Corrigé de l'examen de mécanique générale Janvier 2011

I. Etude statique (13 points)

1)

- **Action de l'air sur le rotor au point P**
- **Action du pignon 31 sur la roue (2) au point K**
- **Action de la liaison linéaire annulaire au point A**
- **Action de l'appui plan au point B**

Donc en tout quatre actions mécaniques

2)

- Liaison linéaire annulaire d'axe $(A, \overrightarrow{y_1})$:

$$\{\tau_{lin/2}\}_{R_1}^{A} = \begin{Bmatrix} X_A & 0 \\ 0 & 0 \\ Z_A & 0 \end{Bmatrix}$$

- Liaison appui plan de normal $(B, \overrightarrow{y_1})$: $\{\tau_{plan/2}\}_{R_1}^{B} = \begin{Bmatrix} 0 & L_B \\ Y_B & 0 \\ 0 & N_B \end{Bmatrix}$

3)

Transfert des moments en B

❖ $$\vec{M}_B(\vec{R}_{air/rotor}) = \vec{M}_A(\vec{R}_{air/rotor}) + \overrightarrow{BP} \wedge \vec{R}_{air/rotor}$$

$$= \begin{pmatrix} 0 \\ M \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -(a+b+c) \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ F \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ F \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{M}_B(\vec{R}_{air/rotor}) = \begin{pmatrix} 0 \\ M \\ 0 \end{pmatrix}$$

$$d'ou\ \{\tau_{air/rotor}\}_{R_1}^{B} = \begin{Bmatrix} 0 & O \\ F & M \\ 0 & 0 \end{Bmatrix}$$

❖ $$\vec{M}_B(\vec{R}_{3/2}) = \vec{M}_K(\vec{R}_{3/r2}) + \overrightarrow{BK} \wedge \vec{R}_{3/2}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -C \\ -\frac{D_2}{2} \end{pmatrix} \wedge \begin{pmatrix} F_T \\ -F_T tan\beta \\ F_T \frac{tan\alpha}{cos\beta} \end{pmatrix}$$

$$= \begin{pmatrix} -CF_T \frac{tan\alpha}{cos\beta} - \frac{D_2}{2} F_T tan\beta \\ -\frac{D_2}{2} F_T \\ CF_T \end{pmatrix}$$

$$D'ou\left\{\tau_{\vec{R}_{3/2}}\right\}_{R_1}^{B} = \left\{\begin{matrix} F_T \\ -F_T tan\beta \\ F_T \frac{tan\beta}{cos\beta} \end{matrix}\middle| \begin{matrix} -F_T\left(C\frac{tan\alpha}{cos\beta}+\frac{D_2}{2}tan\beta\right) \\ -\frac{D_2}{2}F_T \\ CF_T \end{matrix}\right\}$$

- ❖ $\vec{M}_B(\vec{R}_{lin/2}) = \vec{M}_A(\vec{R}_{lin/2}) + \overrightarrow{BA} \wedge \vec{R}_{lin/2}$

$$= \begin{pmatrix}0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\-(b+C)\\0\end{pmatrix} \wedge \begin{pmatrix}X_A\\0\\Z_A\end{pmatrix} = \begin{pmatrix}-(b+c)Z_A\\0\\(b+c)X_A\end{pmatrix}$$

$$D'ou\left\{\tau_{\vec{R}_{lin/2}}\right\}_{R_1}^{B} = \left\{\begin{matrix} X_A \\ 0 \\ Z_A \end{matrix}\middle| \begin{matrix} -(b+c)Z_A \\ 0 \\ (b+c)X_A \end{matrix}\right\}$$

❖ $\left\{\tau_{\vec{R}_{plan/2}}\right\}_{R_1}^{B} = \left\{\begin{matrix} 0 \\ Y_B \\ 0 \end{matrix}\middle| \begin{matrix} L_B \\ 0 \\ N_B \end{matrix}\right\}$

4) PFS : $\sum\{\tau\}_B^{R_1} = \{0\}$

D'où $\left\{\begin{matrix} 0 \\ F \\ 0 \end{matrix}\middle| \begin{matrix} O \\ M \\ 0 \end{matrix}\right\} + \left\{\begin{matrix} F_T \\ -F_T tan\beta \\ F_T \frac{tan\beta}{cos\beta} \end{matrix}\middle| \begin{matrix} -F_T\left(C\frac{tan\alpha}{cos\beta}+\frac{D_2}{2}tan\beta\right) \\ -\frac{D_2}{2}F_T \\ CF_T \end{matrix}\right\} + \left\{\begin{matrix} X_A \\ 0 \\ Z_A \end{matrix}\middle| \begin{matrix} -(b+c)Z_A \\ 0 \\ (b+c)X_A \end{matrix}\right\} + \left\{\begin{matrix} 0 \\ Y_B \\ 0 \end{matrix}\middle| \begin{matrix} L_B \\ 0 \\ N_B \end{matrix}\right\} =$

$\left\{\begin{matrix} 0 \\ 0_B \\ 0 \end{matrix}\middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right\}$

Théorème de la résultante :

$0 + F_T + X_A + 0 = 0$ **(1)**

$F - F_T tan\beta + 0 + Y_B = 0$ **(2)**

$O + F_T \frac{tan\beta}{cos\beta} + Z_A + 0 = 0$ **(3)**

Théorème du moment :

$O - F_T\left(C\frac{tan\alpha}{cos\beta}+\frac{D_2}{2}tan\beta\right) - (b+c)Z_A + L_B = 0$ **(4)**

$M - \frac{D_2}{2}F_T + 0 + 0 = 0$ **(5)**

$0 + CF_T + (b+c)X_A + N_B = 0$ **(6)**

Résolution :

$D'après\ (5) :\ F_T = \frac{2M}{D_2}$; $AN :\ F_T = 44{,}373\ 10^3 N$

$D'après\ (1) : X_A = -F_T$; $AN : X_A = -44{,}373\ 10^3 N$

$D'après\ (2) :\ Y_B = F_T tan\beta - F$; $AN :\ Y_B = 5682{,}67\ N$

$D'après\ (3) : Z_A = -F_T \frac{tan\beta}{cos\beta}\ ; AN: Z_A = -16981{,}59\ N$

$D'après\ (4) : L_B = (b+c)Z_A + F_T \left(C\frac{tan\alpha}{cos\beta} + \frac{D_2}{2} tan\beta\right)\ ; AN: L_B = 859272{,}7\ N.mm$

$D'après\ (6) : N_B = -CF_T - (b+c)X_A\ ; AN: N_B = 1597428\ N$

Etude Cinématique (7 points):

1)

a) $\vec{\Omega}_{2/1} = \vec{\Omega}_{R_2/R_1} = ?$

$$\vec{\Omega}_{2/1} = \dot{\boldsymbol{\theta}}_2 \vec{\boldsymbol{y}}_1 = \dot{\boldsymbol{\theta}}_2 \vec{\boldsymbol{y}}_2$$

$$\vec{V}_{k2/1} = \frac{d}{dt}(\overrightarrow{AK_2})_{R1} = \frac{d}{dt}\left(b\vec{y_1} + \frac{D_2}{2}\vec{z_2}\right)_{R1}$$

$$= b\frac{d}{dt}(\vec{y_1})_{R1} + \frac{D_2}{2}\frac{d}{dt}(\vec{z_2})_{R1}$$

$$= \frac{D_2}{2}\left[\frac{d}{dt}(\vec{z_2})_{R2} + \vec{\Omega}_{2/1} \wedge \vec{z_2}\right] = \frac{D_2}{2}[\dot{\theta}_2\vec{y_1} \wedge \vec{z_2}] = \frac{D_2}{2}[\dot{\theta}_2\vec{y_2} \wedge \vec{z_2}] =$$

$$\frac{D_2}{2}\begin{pmatrix}0\\ \dot{\theta}_2\\ 0\end{pmatrix} \wedge \begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{D_2}{2}\begin{pmatrix}\dot{\theta}_2\\0\\0\end{pmatrix} = \frac{D_2}{2}\dot{\theta}_2\vec{x}_2$$

$$\boxed{\vec{V}_{k2/1} = \frac{D_2}{2}\dot{\theta}_2\vec{x}_2}$$

2)

a) $\vec{\Omega}_{3/1} = \vec{\Omega}_{R_3/R_1} = ?$

$$\boxed{\vec{\Omega}_{3/1} = -\dot{\theta}_3\vec{y}_1 = -\dot{\theta}_3\vec{y}_3}$$

b) $\vec{V}_{k3/1} = \frac{d}{dt}(\overrightarrow{AK_3})_{R1} = \frac{d}{dt}\left(b\vec{y_1} - \left(\frac{D_2}{2} + \frac{D_{31}}{2}\right)\vec{z_1} + \frac{D_{31}}{2}\vec{z_3}\right)_{R1}$

$$= \frac{D_{31}}{2}\frac{d}{dt}(\vec{z_3})_{R1} = \frac{D_{31}}{2}\left[\frac{d}{dt}(\vec{z_3})_{R3} + \vec{\Omega}_{3/1} \wedge \vec{z_3}\right]$$

$$= \frac{D_{31}}{2}\begin{pmatrix}0\\ \dot{\theta}_3\\ 0\end{pmatrix} \wedge \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\vec{V}_{k3/1} = \frac{D_{31}}{2}\begin{pmatrix}\dot{\theta}_3\\0\\0\end{pmatrix}$$

$$\boxed{\vec{V}_{k3/1} = -\frac{D_{31}}{2}\dot{\theta}_3\vec{x_3}}$$

3) $\left\|\vec{V}_{k2/1}\right\| = \left\|\vec{V}_{k3/1}\right\|$

$$\frac{D_2}{2}\dot{\theta}_2 = \frac{D_{31}}{2}\dot{\theta}_3 \longrightarrow \boxed{\dot{\theta}_2 = \frac{D_{31}}{2}\dot{\theta}_3}$$

4) $\dot{\theta}_4 = \frac{D_{32}}{D_4}\,\dot{\theta}_3 \Longrightarrow \dot{\theta}_4 = \frac{D_{32}}{D_4}\frac{D_2}{D_{31}}\,\dot{\theta}_2$