Opérations sur les racines carrées

Exercice 1

Simplifier l'écriture de :

$A = \sqrt{3} \times \sqrt{6}$

$B = \sqrt{5} \times \sqrt{20}$

$C = \sqrt{12} \times \sqrt{27}$

$D = \sqrt{3} \times \sqrt{6} \times \sqrt{8}$

$E = \sqrt{98} \times \sqrt{50}$

$F = \sqrt{15} \times \sqrt{135}$

Exercice 2

Simplifier l'écriture de :

$A = 2\sqrt{2} \times \sqrt{50}$

$B = \sqrt{15} \times 3 \times \sqrt{10}$

$C = 2\sqrt{27} \times 6\sqrt{3}$

$D = 3\sqrt{2} \times \sqrt{8} \times 2\sqrt{2}$

Exercice 3

Simplifier les sommes suivantes:

$A = 5\sqrt{3} - 5\sqrt{28} - \sqrt{7}$

$B = 7\sqrt{2} - \sqrt{18} - 2\sqrt{32}$

$C = 2\sqrt{12} - 4\sqrt{75} + 3\sqrt{27}$

$D = \sqrt{8} - \sqrt{32} + \sqrt{50}$

Exercice 4

Ecrire les nombres suivants sans le symbole racine carré au dénominateur.

Exemple : $\frac{\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$

1 $\frac{\sqrt{28}}{\sqrt{21}}$

2 $\frac{3\sqrt{6}}{2\sqrt{3}}$

3 $\frac{4\sqrt{10}}{5\sqrt{2}}$

4 $\frac{2-\sqrt{3}}{3\sqrt{6}}$

5 $\frac{\sqrt{2}-2\sqrt{5}}{3\sqrt{10}}$

6 $\frac{10\sqrt{6}-3\sqrt{10}}{2\sqrt{15}}$

Exercice 5

Écrire ces expressions sous la forme $a\sqrt{b}$ où b est un entier naturel le plus petit possible et a un entier relatif.

$$A = 5\sqrt{48} - 2\sqrt{75} + 7\sqrt{108}$$
$$B = 3\sqrt{20} + 2\sqrt{45} - 6\sqrt{245}$$
$$C = -5\sqrt{28} + 3\sqrt{112} + 2\sqrt{175}$$

Correction

Exercice 1

$$A = \sqrt{3} \times \sqrt{6} = \sqrt{3 \times 6} = \sqrt{3 \times 3 \times 2} = 3\sqrt{2}$$
$$B = \sqrt{5} \times \sqrt{20} = \sqrt{5 \times 20} = \sqrt{100} = 10$$

$$\begin{aligned} C &= \sqrt{12} \times \sqrt{27} \\ &= \sqrt{12 \times 27} \\ &= \sqrt{3 \times 4 \times 9 \times 3} \\ &= 3 \times 2 \times 3 \\ &= 18 \end{aligned}$$

$$\begin{aligned} D &= \sqrt{3} \times \sqrt{6} \times \sqrt{8} \\ &= \sqrt{3 \times 6 \times 8} \\ &= \sqrt{3 \times 3 \times 2 \times 2 \times 4} \\ &= 3 \times 2 \times 2 \\ &= 12 \end{aligned}$$

$$\begin{aligned} E &= \sqrt{98} \times \sqrt{50} \\ &= \sqrt{49 \times 2} \times \sqrt{25 \times 2} \\ &= 7\sqrt{2} \times 5\sqrt{2} \\ &= 35 \times 2 \\ &= 70 \end{aligned}$$

$$\begin{aligned} F &= \sqrt{15} \times \sqrt{135} \\ &= \sqrt{15} \times \sqrt{9 \times 15} \\ &= 15 \times 3 \\ &= 45 \end{aligned}$$

Exercice 2

$$\begin{aligned} A &= 2\sqrt{2} \times \sqrt{50} \\ &= 2\sqrt{2 \times 50} \\ &= 2\sqrt{100} \\ &= 2 \times 10 \\ &= 20 \end{aligned}$$

$$\begin{aligned} B &= \sqrt{15} \times 3 \times \sqrt{10} \\ &= \sqrt{3 \times 5} \times 3 \times \sqrt{2 \times 5} \\ &= 3 \times 5\sqrt{3 \times 2} \\ &= 15\sqrt{6} \end{aligned}$$

$$\begin{aligned} C &= 2\sqrt{27} \times 6\sqrt{3} \\ &= 12\sqrt{27 \times 3} \\ &= 12\sqrt{81} \\ &= 12 \times 9 \\ &= 108 \end{aligned}$$

$$\begin{aligned} D &= 3\sqrt{2} \times \sqrt{8} \times 2\sqrt{2} \\ &= 6 \times 2 \times \sqrt{8} \\ &= 12\sqrt{4 \times 2} \\ &= 24\sqrt{2} \end{aligned}$$

Exercice 3

$$\begin{aligned} A &= 5\sqrt{3} - 5\sqrt{28} - \sqrt{7} \\ &= 5\sqrt{3} - 5\sqrt{4\times 7} - \sqrt{7} \\ &= 5\sqrt{3} - 5\sqrt{4}\times\sqrt{7} - \sqrt{7} \\ &= 5\sqrt{3} - 5\times 2\times\sqrt{7} - \sqrt{7} \\ &= 5\sqrt{3} - 10\sqrt{7} - \sqrt{7} \\ &= 5\sqrt{3} - 11\sqrt{7} \end{aligned}$$

$$\begin{aligned} B &= 7\sqrt{2} - \sqrt{18} - 2\sqrt{32} \\ &= 7\sqrt{2} - \sqrt{9\times 2} - 2\sqrt{16\times 2} \\ &= 7\sqrt{2} - \sqrt{9}\times\sqrt{2} - 2\sqrt{16}\times\sqrt{2} \\ &= 7\sqrt{2} - 3\times\sqrt{2} - 2\times 4\times\sqrt{2} \\ &= 7\sqrt{2} - 3\sqrt{2} - 8\sqrt{2} \\ &= -4\sqrt{2} \end{aligned}$$

$$\begin{aligned} C &= 2\sqrt{12} - 4\sqrt{75} + 3\sqrt{27} \\ &= 2\sqrt{4\times 3} - 4\sqrt{25\times 3} + 3\sqrt{9\times 3} \\ &= 2\sqrt{4}\times\sqrt{3} - 4\sqrt{25}\times\sqrt{3} + 3\sqrt{9}\times\sqrt{3} \\ &= 2\times 2\times\sqrt{3} - 4\times 5\times\sqrt{3} + 3\times 3\times\sqrt{3} \\ &= 4\sqrt{3} - 20\sqrt{3} + 9\sqrt{3} \\ &= -7\sqrt{3} \end{aligned}$$

$$\begin{aligned} D &= \sqrt{8} - \sqrt{32} + \sqrt{50} \\ &= \sqrt{4\times 2} - \sqrt{16\times 2} + \sqrt{25\times 2} \\ &= \sqrt{4}\times\sqrt{2} - \sqrt{16}\times\sqrt{2} + \sqrt{25}\times\sqrt{2} \\ &= 2\times\sqrt{2} - 4\times\sqrt{2} + 5\times\sqrt{2} \\ &= 2\sqrt{2} - 4\sqrt{2} + 5\sqrt{2} \\ &= 3\sqrt{2} \end{aligned}$$

Exercice 4

1.

$$\begin{aligned}\frac{\sqrt{28}}{\sqrt{21}} &= \frac{\sqrt{4}\times\sqrt{7}}{\sqrt{3}\times\sqrt{7}}\\ &= \frac{\sqrt{4}}{\sqrt{3}}\\ &= \frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}\\ &= \frac{2\sqrt{3}}{3}\end{aligned}$$

2.

$$\begin{aligned}\frac{3\sqrt{6}}{2\sqrt{3}} &= \frac{3\sqrt{3}\times\sqrt{2}}{2\sqrt{3}}\\ &= \frac{3\sqrt{2}}{2}\end{aligned}$$

3.

$$\begin{aligned}\frac{4\sqrt{10}}{5\sqrt{2}} &= \frac{4\sqrt{5}\times\sqrt{2}}{5\sqrt{2}}\\ &= \frac{4\sqrt{5}}{5}\end{aligned}$$

4.

$$\begin{aligned}\frac{2-\sqrt{3}}{3\sqrt{6}} &= \frac{2-\sqrt{3}}{3\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}\\ &= \frac{2\sqrt{6}-\sqrt{3\times 6}}{3\times 6}\\ &= \frac{2\sqrt{6}-3\sqrt{2}}{18}\end{aligned}$$

5.

$$\begin{aligned}\frac{\sqrt{2}-2\sqrt{5}}{3\sqrt{10}} &= \frac{\sqrt{2}-2\sqrt{5}}{3\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} \\ &= \frac{\sqrt{20}-2\sqrt{50}}{3\times 10} \\ &= \frac{\sqrt{4\times 5}-2\sqrt{25\times 2}}{30} \\ &= \frac{2\sqrt{5}-10\sqrt{2}}{30} \\ &= \frac{\sqrt{5}-5\sqrt{2}}{15}\end{aligned}$$

6.

$$\begin{aligned}\frac{10\sqrt{6}-3\sqrt{10}}{2\sqrt{15}} &= \frac{10\sqrt{6}-3\sqrt{10}}{2\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}} \\ &= \frac{10\sqrt{90}-3\sqrt{150}}{2\times 15} \\ &= \frac{30\sqrt{10}-15\sqrt{6}}{30} \\ &= \frac{2\sqrt{10}-\sqrt{6}}{2}\end{aligned}$$

Exercice 5

$$\begin{aligned}A &= 5\sqrt{48}-2\sqrt{75}+7\sqrt{108} \\ &= 5\sqrt{3\times 16}-2\sqrt{3\times 25}+7\sqrt{3\times 36} \\ &= 5\times 4\sqrt{3}-2\times 5\sqrt{3}+7\times 6\sqrt{3} \\ &= 20\sqrt{3}-10\sqrt{3}+42\sqrt{3} \\ &= 52\sqrt{3}\end{aligned}$$

$$\begin{aligned}B &= 3\sqrt{20}+2\sqrt{45}-6\sqrt{245} \\ &= 3\sqrt{4\times 5}+2\sqrt{9\times 5}-6\sqrt{49\times 5} \\ &= 3\times 2\sqrt{5}+2\times 3\sqrt{5}-6\times 7\sqrt{5} \\ &= 6\sqrt{5}+6\sqrt{5}-42\sqrt{5} \\ &= -30\sqrt{5}\end{aligned}$$

$$\begin{aligned}C &= -5\sqrt{28}+3\sqrt{112}+2\sqrt{175} \\ &= -5\sqrt{4\times 7}+3\sqrt{16\times 7}+2\sqrt{25\times 7} \\ &= -5\times 2\sqrt{7}+3\times 4\sqrt{7}+2\times 5\sqrt{7} \\ &= -10\sqrt{7}+12\sqrt{7}+10\sqrt{7} \\ &= 12\sqrt{7}\end{aligned}$$